

Geometry I, 2006 : Coursework 9

1. [unassessed] Do exercise 2 on page 71 of the course text, and check your answers in the back of the book. [Note that the course text writes st for $s \circ t$.]

2. (i) [2 marks] Define what it means for a function $t : \mathbb{R}^n \rightarrow \mathbb{R}^m$ to be a *linear transformation*.

(ii) [4 marks] Is the function $t : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $t \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a + 2b \end{pmatrix}$ a linear transformation? Why or why not?

Is the function $t : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $t \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^2 \\ b \end{pmatrix}$ a linear transformation? Why or why not?

3. [4 marks] Let $s : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation with

$$s \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, s \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \text{ and let } t : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \text{ be a linear transformation with}$$
$$t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Compute the matrices representing s , t , and $s \circ t$.

4. [3 marks] Suppose $s : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $t : \mathbb{R}^n \rightarrow \mathbb{R}^m$ are linear transformations. Define the function $s + t : \mathbb{R}^n \rightarrow \mathbb{R}^m$ by $(s + t)(u) = s(u) + t(u)$ for all $u \in \mathbb{R}^n$.

Prove that if s is represented by the matrix A and t is represented by the matrix B , then $(s + t)(u) = (A + B)u$ for all $u \in \mathbb{R}^n$. [This shows that $s + t$ is a linear transformation represented by $A + B$.]

5. [3 marks] Suppose $s : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $t : \mathbb{R}^n \rightarrow \mathbb{R}^n$ are linear transformations, with s represented by the matrix A and t represented by the matrix B . Prove that if $B = A^{-1}$ then $(s \circ t)(u) = u$ for all $u \in \mathbb{R}^n$.

6. Recall that S_θ is the matrix representing a reflection (in the (x, y) -plane) in the line through the origin at (counterclockwise) angle θ from the x -axis.

(i) [2 marks] Prove that $(S_\theta)^{-1} = S_\theta$.

(ii) [2 marks] Prove that $(S_{\theta_1} S_{\theta_2} S_{\theta_3})^{-1} = S_{\theta_3} S_{\theta_2} S_{\theta_1}$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 7th December.

Leonard Soicher, 30 November 2006